# A few sample problems for inferential statistics

### Problems.

1. Suppose  $X_1, \ldots, X_{100}$  are i.i.d random variables which have uniform distribution on [a-2, a+2], where a is unknown. Suppose the random sample produces sample mean equal to 3.

Compute a 95% confidence interval for a.

- 2. In a mythical national survey, 225 students are randomly selected from those taking calculus, and asked if calculus is their favorite subject. 100 students reply that calculus is their favorite subject. Give a 95% confidence interval for the proportion of all students taking calculus who consider it their favorite subject.
- 3. Suppose in a random sample of 225 undergraduate men at UMD that the average best (highest weight) bench press is 150 pounds, with sample standard deviation of 20 pounds. Compute a 95% confidence interval for the average best bench press for for UMD undergraduate men.

Solutions to the problems are on the following pages.

### Solutions.

1. Suppose  $X_1, \ldots, X_{100}$  are i.i.d random variables which have uniform distribution on [a-2, a+2], where a is unknown. Suppose the random sample produces sample mean equal to 3.

Compute a 95% confidence interval for a.

### 1. SOLUTION

A random variable with uniform distribution on [a-2, a+2] has mean  $\mu = a$ . So, a confidence interval for  $\mu$  is a confidence interval for a. Because n = 100 is large, the confidence interval provided by the Central Limit Theorem applies:

$$\left(\overline{X} - 1.96 \frac{\sigma}{\sqrt{n}}, \ \overline{X} + 1.96 \frac{\sigma}{\sqrt{n}}\right)$$

A random variable with uniform distribution on [a-2, a+2] has standard deviation  $\sigma = 4/\sqrt{12}$ . Our sample mean is 3. Substituting, we get

$$\left(3 - (1.96) \frac{(4/\sqrt{12})}{\sqrt{100}}, \ 3 + (1.96) \frac{(4/\sqrt{12})}{\sqrt{100}}\right) \\
= (2.73, \ 3.27) \ .$$

2. In a mythical national survey, 225 students are randomly selected from those taking calculus, and asked if calculus is their favorite subject. 100 students reply that calculus is their favorite subject. Give a 95% confidence interval for the proportion of all students taking calculus who consider it their favorite subject.

## **SOLUTION**

We will plug into the 95% confidence interval formula for population proportion,

$$\left(\widehat{p} - 1.96 \frac{\sqrt{\widehat{p}(1-\widehat{p})}}{\sqrt{n}}, \ \widehat{p} + 1.96 \frac{\sqrt{\widehat{p}(1-\widehat{p})}}{\sqrt{n}}\right)$$

Here  $\hat{p} = 100/225 = 20/45 = 4/9$  and n = 225, so the interval is

$$= \left(4/9 - 1.96 \frac{\sqrt{(4/9)(5/9)}}{\sqrt{225}}, \ 4/9 + 1.96 \frac{\sqrt{(4/9)(5/9)}}{\sqrt{225}}\right)$$

$$= \left(4/9 - 1.96 \frac{\sqrt{20}}{(9)(15)}, \ 4/9 + 1.96 \frac{\sqrt{20}}{(9)(15)}\right)$$

$$\approx (.38, .51)$$

3. Suppose in a random sample of 225 undergraduate men at UMD that the average best (highest weight) bench press is 150 pounds, with sample standard deviation of 20 pounds. Compute a 95% confidence interval for the average best bench press for for UMD undergraduate men.

### SOLUTION

We use for the interval the formula

$$\left(\overline{X} - 1.96 \frac{s}{\sqrt{n}}, \ \overline{X} + 1.96 \frac{s}{\sqrt{n}}\right)$$

Here the sample mean is 150 and s=20. So the desired 95% confidence interval, in pounds, for the average best bench press of UMD undergraduate men is

$$\left(150 - 1.96 \frac{20}{\sqrt{225}}, \ 150 + 1.96 \frac{20}{\sqrt{225}}\right)$$

$$= \left(150 - 1.96(.8), \ 150 + 1.96(.8)\right)$$

$$= (48.4, 51.6).$$